Unemployment, Fixed Exchange Rate, and Money Supply*

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I. INTRODUCTION

Mundell's famous paper [3] was constructed on the simpleassumptions that the degree of the mobility of capital is extremely high (the assumption of perfect mobility of capital), and that the country under consideration is too small to influence foreign incomes or the world level of interest rates (the small country assumption). And Mundell [4] also generalized the results by taking into account repercussion effects and abandoning the assumption that the country pursuing the policy is small.

Numerous models along these lines such as in R. N. Cooper [8] have been developed, but these are so complicated that it is difficult for us to analyse the results using the graphical technique.

Recently, A. K. Swoboda & R. Dornbusch [1] extend a simple model which takes into account goods and services market to a general one which takes into account goods and services market, money market, and the balance of payments equilibrium. One of their conclusions is as follows:

> In particular, the degree of capital mobility is seen not to affect the impact of money-supply disturbances after full adjustment has taken place; the importance of capital mobility resides in its role in speeding up the process of adjustment. One important corollary of this proposition is that the monetary-policy results derived under the assumption of perfect capital

mobility do not, as was previously thought, constitute an exception but the general rule. 1/

The purpose of this paper is to examine the mathematical backgrounds of the above quoted conclusion and to show Swoboda & Dornbusch [1] treat a special case of the models shown below.

In what follows, we will analyse the general case under (A) Capital Immobility and (B) Capital Mobility, and consider the effects of the following four policies using the mathematical technique as well as graphical technique.

(i) A redistribution of money from the home country to a foreign country which denoted by dM_0

(ii) A money supply increase by the home country which denoted by $d\bar{M}$

(iii) An autonomous increase in expenditure in the home country which denoted by $d\bar{I}$

(iv) An income transfer from the home country to a foreign country which denoted by $d\tau$

II. TWO SYSTEMS UNDER FIXED EXCHANGE RATE AND UNEMPLOYMENT

We have two sets of equations. One is to analyse the state of the quasi-equilibrium point, the other is to examine the state of the long-run equilibrium point. The basic model consists of five equations.

(1) $Y=E(Y, r)+T(Y, Y')+\bar{I}-\tau$

(2) $Y'=E'(Y', r')-T(Y, Y')+\tau$

(3) $\bar{M}+\bar{I}+T(Y, Y')+F(r-r')-M_0-\tau=L(Y, r)$

(4) $\overline{M}'-T(Y, Y')-F(r-r')+M_0+\tau=L'(Y', r')$

(5) $B=T(Y, Y')+F(r-r')-M_0-\tau$,

where we assume that the fixed exchange rate is equal to unity, and unemployment prevails,[2] Y stands for national output, r for the rate of interest, E for domestic expenditures (absorption), T for the trade surplus of the home country, F for capital inflow to the home country, B for the balance of payments of the home country, L for the demand for money, $\overline{M}$ for the home country's money supply, $\bar{I}$ for the debt-financed autonomous expenditures of the home country, τ for income transfer from the home country to a foreign country.

Equation (1) expresses the equilibrium condition of the goods and services market of the home country. Equation (2) is the equilibrium condition of the goods and services market of the foreign country. Equation (3), (4) show the monetary equilibrium conditions of the home country and the foreign country respectively.[3] Equation (5) defines the balance of payments of the home country. We can solve these five equations for the five unknown variables, Y, Y', r, r', B.

Combining equation (3) and (4), and setting $B=0$, we obtain the latter model consisting of four equations (1)' to (4)' which show the state of the final equilibrium point.

(1)' $Y=E(Y, r)+T(Y, Y')+\bar{I}-\tau$

(2)' $Y'=E'(Y', r')-T(Y, Y')+\tau$

(3)' $\overline{M}+\overline{M}'+\bar{I}=L(Y, r)+L'(Y', r')$

(4)' $T(Y, Y')+F(r-r')-M_0-\tau=0$

Differenciating totally two sets of equations, we obtain equa-

tions (6) and (6)′ respectively.

$$(6)\quad \begin{pmatrix} s+m & -m' & -E_r & 0 & 0 \\ -m & s'+m' & 0 & -E'_{r'} & 0 \\ -(m+L_Y) & m' & F'-L_r & -F' & 0 \\ m & -(m'+L'_{Y'}) & -F' & F'-L'_{r'} & 0 \\ m & -m' & -F' & F' & 1 \end{pmatrix} \begin{pmatrix} dY \\ dY' \\ dr \\ dr' \\ dB \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ -1 \end{pmatrix} dM_0 + \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} d\bar{M} + \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} d\bar{I} + \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} d\tau$$

$$(6)'\quad \begin{pmatrix} s+m & -m' & -E_r & 0 \\ -m & s'+m' & 0 & -E'_{r'} \\ L_Y & L'_{Y'} & L_r & L'_{r'} \\ -m & m' & F' & -F' \end{pmatrix} \begin{pmatrix} dY \\ dY' \\ dr \\ dr' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} dM_0 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} d\bar{M}$$

$$+ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} d\bar{I} + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} d\tau$$

where we set $-m=\partial T/\partial Y=T_Y$ $(0<m<1)$, $m'=T_{Y'}(0<m'<1)$, $s=1-E_Y$, $s'=1-E'_{Y'}(0<s, s'<1)$. We assume, $0<E_Y<1$, $E_r<0$, $L_r<0$, $F'=dF/d(r-r')>0$ and these conditions adopt also to primed variables.

The value of the determinants of the coefficient matrix of the lefthand side of equations (6) and (6)′, Δ_1, for equation (6) and Δ_2 for equation (6)′, are both assumed to be positive.

(7) $\Delta_1>0$

(7)′ $\Delta_2>0$

III. THE MONEY REDISTRIBUTION EFFECT FROM THE HOME TO THE FOREIGN COUNTRY

Taking the first parameter of equations (6) and (6)′ into consideration as well as equations (7) and (7)′, we obtain following results.

(8) $dY/dM_o \gtreqless 0$, $dY'/dM_o \gtreqless 0$, $dr/dM_o > 0$, $dr'/dM_o < 0$, $dB/dM_o < 0$

(8)′ $dY/dM_o \gtreqless 0$, $dY'/dM_o \gtreqless 0$, $dr/dM_o > 0$, $dr'/dM_o < 0$,

where equation (8) refers to the quasi-equilibrium point, and equation (8)′ refers to the long-run equilibrium point respectively.

To make the graphical analysis easily, we consider two cases, (A) $F'=0$ which we call the case of the immobility of capital and (B) $F'>0$ which we call the mobility of capital case.

(A) The capital immobility case ($F'=0$)

Setting $F'=0$ in equation (6) and (6)′, and supposing $(dY'/dM_o)/$

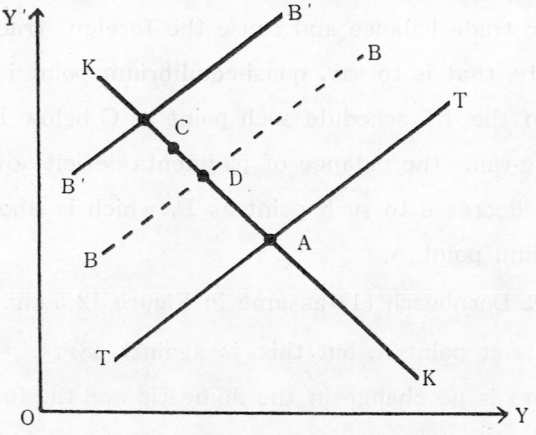

Figure 1.

$(dY'/dM_0) = (dY'/dY)_{KK} < 0$, [4] we can draw the locus of Y and Y' when the money is redistributed from the home to the foreign country in Figure 1.

In Figure 1, TT depicts the trade balance schedule, the upper side of the TT is the region of the home country's trade surplus, the lower side of the TT represents the region of the home country's trade deficit. The KK line refers to the money redistribution schedule at constant money supply. The BB line illustrates the balance of payments equilibrium condition including the money redistribution effect. Point A shows the initial equilibrium position of the system.

The money redistribution from the home to the foreign country increases the money supply of the foreign country, which makes the foreign interest rate lower, decreases the money supply of the home country which raise the domestic interest rate. The interest rate change makes the domestic output decrease, and increases the foreign output. Such income change will improve the domestic trade balance and cause the foreign trade balance to deteriorate, that is to say, quasi-equilibrium point is in the upper region of the TT schedule such point as C below B'B'.

In the long-run, the balance of payments deficits of the home country will decrease to such point as D, which is above the initial equilibrium point A.

Swoboda & Dornbusch [1] assume in Figure 12.5 that the final equilibrium is at point A, but this is against (8)'. [5] Becasue at point A, there is no change in the domestic and the foreign interest rates.

(B) The capital mobility case ($F'>0$)

Swoboda & Dornbusch [1] examine this case in Figure 12.8 which corresponds to the money redistribution from the foreign country to the home country. Here, we consider the same situation as case (A) that shows the money redistribution from the home country to the foreign country.

In Figure 12.8 Swoboda & Dornbusch assume that the final equilibrium will be attained at point A, but this is wrong because of the following reasons.[6] We know from equation (8)′ that the final equilibrium point is somewhere between point A and point D in Figure 1, and that the higher the degree of capital mobility (F' becomes larger), the nearer the final equilibrium position to point A.[7]

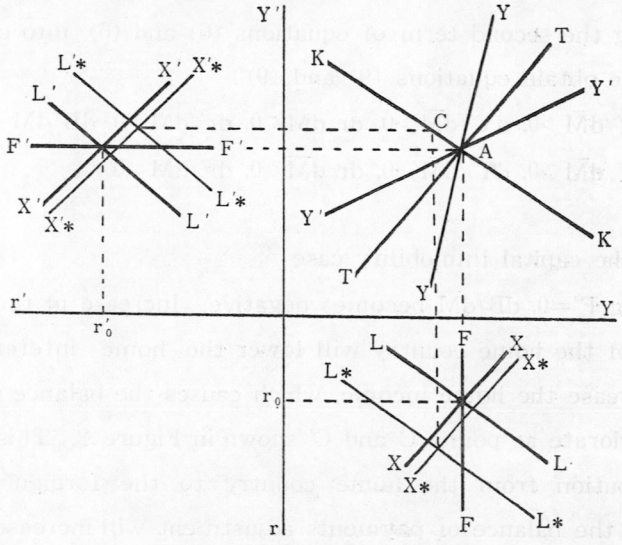

FIGURE 12.5

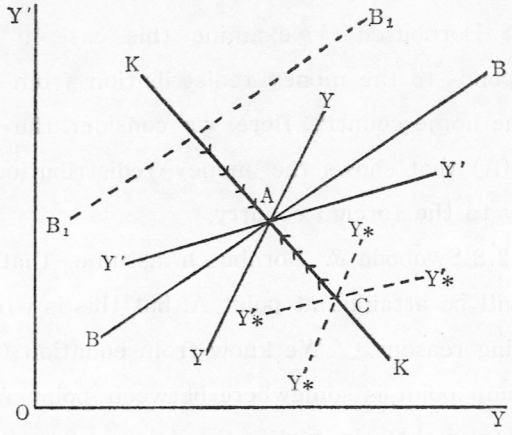

FIGURE 12.8

IV. THE EFFECT OF MONEY SUPPLY OF HOME COUNTRY

Taking the second term of equations (6) and (6)′ into consideration, we obtain equations (9) and (9)′.

(9) $dY/d\bar{M}>0$, $dY'/d\bar{M}>0$, $dr/d\bar{M}<0$, $dr'/d\bar{M}\gtreqless 0$, $dB/d\bar{M}\gtreqless 0$

(9)′ $dY/d\bar{M}>0$, $dY'/d\bar{M}>0$, $dr/d\bar{M}<0$, $dr'/d\bar{M}<0$

(A) The capital immobility case

Setting $F'=0$, $dB/d\bar{M}$ becomes negative. Increase in the money supply of the home country will lower the home interest rate, and increase the home income, which causes the balance of trade to deteriorate as points C and C′ shown in Figure 2. This money redistribution from the home country to the foreign country through the balance of payments adjustment will increase foreign income along K_*K_* schedule up to the final equilibrium point D.

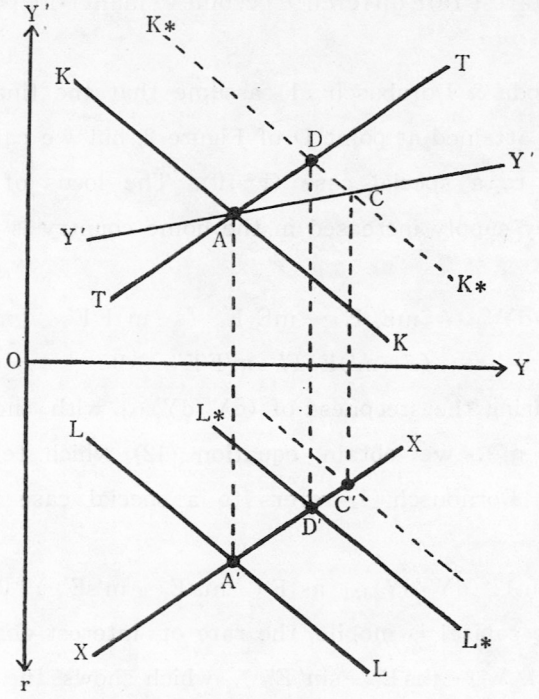

Figure 2.

(B) The capital mobility case ($F'>0$)

Swoboda & Dornbusch [1] analyse this case in Figure 12.6, but the results differ from that of mathematical analysis in two points.

(i) From equation (9), we can compute the value of $d(r-r')/d\overline{M}$, which shows the direction of interest rate change at point C in Figure 2.

(10) $d(r-r')/d\overline{M}=(1/\Delta_1)[(ss'+sm'+ms')L_r$

$+(sm'+sL'_{Y'}+mL'_{Y'})E'_{r'}+mL'_{Y'}E_r-sE_r\,] \gtrless 0$

Contrary to equation (10), Swoboda & Dornbusch [1] assume

that the interest rate difference becomes smaller at point C than at point A.[8]/

(ii) Swoboda & Dornbusch [1] assume that the final equilibrium point is attained at point D of Figure 2, but we can show that this refers to a special case ($F'=0$). The locus of Y and Y' when money supply increased in the home country is given by equation (11).

(11) $(dY'/dY)_{d\bar{M}}=[mE_r E'_{r'}-mE_rF'-(s+m)F'E'_{r'}]/[m'E_r E'_{r'}-(s'+m')E_r F'-m'F'E'_{r'}]>0$

By comparing the steepness of $(dY'/dY)_{d\bar{M}}$ with the steepness of TT (m/m'), we obtain equation (12) which tells us that Swoboda & Dornbusch [1] refers to a special case when $F'=0$ is supposed.

(12) $(m/m')\gtrless(dY'/dY)_{d\bar{M}}$ as $F'(-ms'E_r+m'sE'_{r'})\gtrless 0$

When the capital is mobile, the rate of interest change by $d(r-r')/d\bar{M}=(1/\Delta_2)(-ms'E_r+sm'E'_{r'})$, which shows the same sign as equation (12) (when the capital mobile perfectly, $dr/d\bar{M}=dr'/d\bar{M}$ hold).

The above results are drawn in Figure 3. The broken line AM' and AM'' correspond to $m/m'>(dY'/dY)_{d\bar{M}}$ and $m/m'<(dY'/dY)_{d\bar{M}}$ respectively.

And also points D' and D'' on K_*K_* schedule are the final equilibrium. At point D', there prevail trade deficit and capital surplus that make the balance of payments equal to zero. At point D'', there exist trade surplus and capital deficit simultaneously. We can also find that the final equilibrium point D is a special case which corresponds to the capital immobility case.[9]/

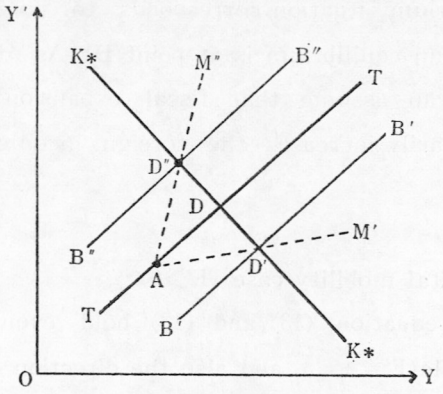

Figure 3.

V. THE EFFECT OF THE DOMESTIC FISCAL EXPANSION

Now, let us consider the effect of the domestic fiscal expansion which accompany the increased money supply of equal amounts (debt-financed fiscal expansion).

Taking the third parameter $d\bar{I}$ of equations (6) and (6)′ into consideration as well as equations (7) and (7)′, we obtain the following results.

(13) $dY/d\bar{I}>0$, $dY'/d\bar{I}\gtreqless 0$, $dr/d\bar{I}\gtreqless 0$, $dr'/d\bar{I}\gtreqless 0$, $dB/d\bar{I}\gtreqless 0$

(13)′ $dY/d\bar{I}>0$, $dY'/d\bar{I}\gtreqless 0$, $dr/d\bar{I}\gtreqless 0$, $dr'/d\bar{I}\gtreqless 0$

(A) The capital immobility case ($F'=0$)

In this case, following signs become unique, such as $dY'/d\bar{I}>0$, $dB/d\bar{I}<0$ in equation (13), and $dY'/d\bar{I}>0$, $dr'/d\bar{I}>0$ in equation (13)′. In Figure 12.6, KK schedule shifts to K_*K_* schedule because of debt-financed fiscal expansion.

Quasi-equilibrium situation corresponds to point C, and the state of long-run equilibrium is at point D. As $dY'/d\bar{I}$ becomes positive, we can assume that fiscal expansion in the home country necessarily increases the foreign income under capital immobility.

(B) The capital mobility case ($F'>0$)

In this case, equation (13) and (13)′ hold even if capital is perfectly mobile($F'=+\infty$), and also the direction of interest rate change at the final equilibrium point is umbigious as equation (14) shows, which corresponds to either point E or F in Figure 12.9.

(14) $d(r-r')/d\bar{I}=(1/\Delta_2)(-m'L_YE'_{r'}-msL'_{r'}-mL'_{Y'}E'_{r'}+m'sE'_{r'}$
$-msL_r-ms'E_r)\gtrless 0$

VI. THE EFFECT OF INCOME TRANSFER FROM THE HOME TO THE FOREIGN COUNTRY

Taking the fourth parameter $d\tau$ into consideration in equations (6) and (6)′, we can obtain equations (15) and (15)′.

(15) $dY/d\tau<0$, $dY'/d\tau>0$, $dr/d\tau\gtrless 0$, $dr'/d\tau\gtrless 0$, $dB/d\tau<0$

(15)′ $dY/d\tau<0$, $dY'/d\tau>0$, $dr/d\tau<0$, $dr'/d\tau<0$

(A) The capital immobility case ($F'=0$)

In Figure 12.7, Swoboda & Dornbusch [1] assume that the final equilibrium position is at point C along K_*K_* schedule,[10] but as we have seen it is along KK schedule under constant money stock.

This case is drawn as in Figure 4, where the final equilibrium is attained at point C and the quasi-equilibrium is at point D with

the balance of payments deficit.

(B) The capital mobility case ($F'>0$)

Equations (15) and (15)′ hold for this case even if capital is perfect mobile($F'=+\infty$), and equation (16) holds for capital balance, which shows no unique direction of change in the balance of payments schedule.[11] This concludes that the final equilibrium point is necessarily at point C along KK schedule in Figure 4.[12]

(16) $d(r-r')/d\tau=(1/\Delta_2)(-sL'_{Y'}F'+ss'L'_{r'}+sL'_{Y'}E'_{r'}+s'L_YF'$
$-ss'L_r-sL'_{Y'}E_r)\gtrless 0$

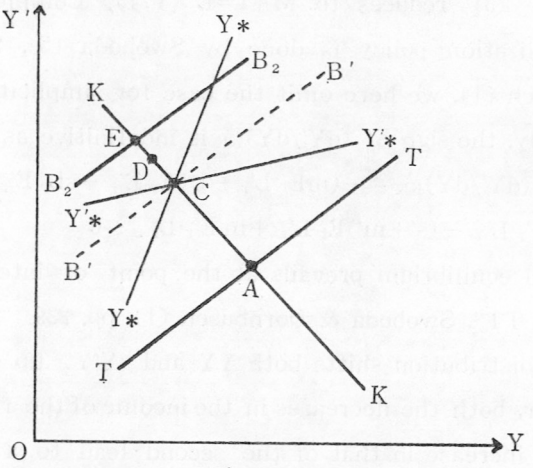

Figure 4.

Footnotes:

* I am deeply indebted to Prof. Akira Takayama for his helpful suggestions. All errors are, of course, my own responsibility. While I was attending the seminor on International Trade and Finance, held at Nagoya City University, Nagoya, Japan, in March 1975, the lecturer, Prof. Akira Takayama of Purdue University, brought our attention upon several issues of critical and theoretical importance in the work of A.K. Swoboda & R. Dornbusch,

"Adjustment, Policy, and Monetary Equilibrium in a Two-Country Model". This gave us a growing interest, and led me into further study of Swoboda & Dornbusch [1].

Thanks are due to the Ministry of Education, science and culture for research support.

1/ Swoboda & Dornbusch [1], pp. 257.

2/ We can analyse unemployment as a special case. For example, setting $\sigma = dY/dP$ (P stands for the price of output) as A. Takayama [2], this case is for $\sigma > 0$.

3/ If the sterilization operation is taken by the home country, equation (3) reduces to $\bar{M} + \bar{I} = L$ (Y, r). Complex analysis of sterilization policy is done by Swoboda [5], Swoboda & Dornbusch [1], we here omit the case for simplicity.

4/ Generally, the sign of $(dY'/dY)_{KK}$ is indefinitive as can be seen below. $(dY'/dY)_{KK} = -(mE_r L'_{r'} - L_Y E_r E'_{r'} - sL_r E'_{r'} - mL_r E'_{r'}) / [-E_r E'_{r'} L'_{Y'} - (s' + m') E_r L'_{r'} + m' E'_{r'} L_r] \gtrless 0$

5/ "General equilibrium prevails at the point of intersection of KK and TT", Swoboda & Dornbusch [1], pp. 239.

"The redistribution shifts both YY and Y'Y' up along KK. However, both the decreases in the income of the first country and the increase in that of the second lead to a deficit for country 2. Reserves flow back from the second to the first country until the initial income levels and distribution of the world money stock are re-established.", Swoboda & Dornbusch [1], pp. 241.

6/ "The reserve flows associated with country 1's deficit will return the system to A, as when capital is immobile inter-

nationally.", Swoboda & Dornbusch [1], pp. 249.

7/ In equation (8)', only denominator Δ_2 includes term F', so Y, Y', r, r' will come to point A as F' becomes larger to infinity.

8/ See footnote 27, and "At the same time, the balance-of-payments schedule shifts up to some position B_1B_1 (not drawn) since at C the interest differential is lower than at A.", Swoboda & Dornbusch [1], pp. 250.

9/ "Final equilibrium, however, must again be at D", Swoboda & Dornbusch [1], pp. 250.

10/ "The transfer's deficit leads to a redistribution of the world money supply towards country 2 and thus to an upward movement along K_*K_*. the locus of monetary equilibrium through D, until full equilibrium is reached at point C.", Swoboda & Dornbusch [1], pp. 245.

11/ Swoboda & Dornbusch [1] assume that $d(r-r')/d\tau<0$ in footnote 31, pp. 252.

12/ "Final equilibrium is reached along KK, either below or above D (the capital immobility equilibrium point) but, in any event, above C, as before.", Swoboda & Dornbusch [1], pp. 253.

References

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〔6〕 Swoboda, "Monetary Policy under Fixed Exchange Rates: Effectiveness, the Speed of Adjustment and Proper Use", *Economica*, Vol. 15, No. 158, May 1973.

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